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Energy-Level Shifts of a One-Electron Atom in Higher-Derivative Gravitational Field Caused by High-Power Laser Pulse

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The gravitational field produced by a high-power laser pulse traveling with a velocity $\upsilon < c$ is found in the framework of higher-derivative theory of gravitation. The gravitational perturbations of the energy levels of a freely falling one-electron atom in the higher-derivative gravitational field are studied. The energy-level shifts of highly excited hydrogen atom are considered, and the influence of the additional forces included in the linearized higher-derivative gravitation on the energe level shifts of the atom is investigated.

KEY WORDS: higher-derivative gravitation; gravitational perturbation; energy-level shift.

1. INTRODUCTION

The general theory of relativity and higher-derivative theories of gravitation predict that gravitation is manifested as curvature of spacetime. This curvature is characterized by the Riemann tensor $R^{\alpha}{}_{\mu\nu\beta}$. The energy levels of an atom placed in a region of curved spacetime will be shifted as a result of the interaction of the atom with the local curvature. Frequency shifts caused by local curvature differ from the usual Doppler, gravitational, and cosmological shifts. The level spacing of the gravitational effect is different from that of the well-known first-order (degenerate) Stark and Zeeman effects. It would be possible to separate the electromagnetic and gravitational perturbation of the spectra (Parker, 1980a,b). Thus, in principle, atomic spectra carry unambiguous information about the local curvature at the position of the atom, and one can envision the possibility of using atomic spectrum to detect possible regions of high curvature.

The gravitational perturbation of the atomic spectrum have been discussed by a number of authors within the context of general relativity. The energy-level

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of a one-electron atom in a general curved spacetime was investigated by Parker (1980a,b). The Dirac Hamiltonian was evaluated in Fermi normal coordinates to first order in the Riemann curvature tensor of an arbitrary spacetime, and the energy shifts of the relativistic $1S_{1/2}$, $2S_{1/2}$, and $2P_{1/2}$ levels, and the nonrelativistic $1S$, $2S$, and $2P$ levels are calculated. The hydrogen atom in certain cosmological spacetimes was discussed by Audretsch and Schäfer (1978). Atoms at rest in the Schwarzschild and Robertson-Walker metrices were studied by Tourrenc and Grossiord (1976). Parker and Pimentel (1982) investigated the perturbations of the energy levels of a freely falling one-electron atom in a curved spacetime, calculated the energy-levels shifts to first order in the Riemann tensor for the relativistic $2P_{3/2}$ levels and the nonrelativistic 3S, 3P, and 3D levels, and discussed the order of magnitude of the energy-level perturbation for highly excited Rydberg atoms.

With the development of the high-intense laser technology, the pulsed laser may presently involve megajoule energies, which intensity may come up to an order of magnitude above 10^{20} W/cm^2. Scully (1979) considered the gravitational field produced by high-power laser pulse and discussed the gravitational deflection and the phase shift of a probe laser pulse propagating in the neighborhood of the high-power laser pulse in general relativity. The energy-level shifts of a one-electron atom in the curved spacetime caused by the strong short-laser pulse were recently studied by Ji *et al.* (1998). It was shown that under the present conditions of high-power laser pulse, the magnitude of the energy-levels shifts of a highly excited hydrogen atom will be observable, and a possible channel of testing general relativity was suggested.

It was pointed out by various authors (Fujii, 1971; Long, 1974; Mikkelson and Newman, 1977), some years ago, that existing experimental data cannot exclude a possibility that Newton's law may be violated at distances less than 10^3 km. From a theoretical point of view, an additional short-range field could be added to the Newtonian component without being detected at large distances. A most likely range of force in which a significant departure from the Newtonian inverse square law of gravitation could have gone undetected would be $\beta^{-1} < 1$ cm or 10 m $\leq \beta^{-1} \leq 1$ km where experimental data are poorest (Fujii, 1971; Mikkelson and Newman, 1977). Such a departure from the inverse square law could be understood in a generalized scalar-tensor theory (Fujii, 1974; O'Hanlon, l972) or higher-derivative theories of gravitation (Stelle, 1978; Xu and Ellis, 1991).

Since the time of Weyl and Eddington, higher-derivative gravitational theories have been discussed by several generations of scientists, and applied to quantum gravity (Stelle, 1977), early cosmology (Barrow and Ottewill, 1983), pure gravitational inflationary model for the universe (Berkin, 1990; Mijic *et al.*, 1986), eliminating the singularities in gravity (Müller, 1985; Treder, 1975), and so on. In this paper, we find the gravitational field produced by a high-intensity short laser pulse in the framework of higher-derivative theory of gravitation, investigate gravitational perturbation of hydrogen spectrum, and compare our results with that in general relativity.

2. HIGHER-DERIVATIVE GRAVITY CAUSED BY HIGH-POWER LASER PULSE

A general action for higher-derivative gravity may be written as

$$S = \int d^4x[\sqrt{-g}(R + aR^2 + bR_{\mu\nu}R^{\mu\nu}) - kL_{\rm m}]$$

where k is Einstein's constant with $k = 8\pi G/c^2$ (G being Newton's constant), a and b are two parameters, $L_{\rm m}$ is the Lagrangian density of the matter fields involved, $R_{\mu\nu}$ is the Ricci tensor and R is the trace of Ricci tensor.

The variation of the above action with respect to the metric yields the higher-derivative field equations (Dewitt, 1965; Xu and Ellis, 1991)

$$G_{\mu\nu} = G^{(E)}_{\mu\nu} + aG^{(1)}_{\mu\nu} - bG^{(2)}_{\mu\nu} = kT_{\mu\nu}, \tag{2.1}$$

where

$$G^{(E)}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R,$$

$$G^{(1)}_{\mu\nu} = 2R_{;\mu;\nu} - 2g_{\mu\nu}R_{;\sigma}{}^{;\sigma} + 2RR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R^2,$$

$$G^{(2)}_{\mu\nu} = g_{\mu\nu}R^{\sigma\rho}{}_{;\sigma;\rho} + g^{\sigma\rho}(R_{\mu\nu;\sigma;\rho} - R_{\mu\sigma;\nu;\rho} - R_{\nu\sigma;\mu;\rho})$$
$$-2R_{\mu\sigma}R^{\sigma}_{\nu} + \frac{1}{2}g_{\mu\nu}R_{\sigma\rho}R^{\sigma\rho}.$$

$T_{\mu\nu}$ is the energy–momentum tensor of the matter fields. A semicolon denotes covariant derivative. The gravitational field is supposed to be weak. So we put

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{2.2}$$

where, in our convention, the flat spacetime metric is given by $\eta_{\mu\nu} = \mathrm{diag}(c^2, -1, -1, -1)$, and $h_{\mu\nu}$ characterizes the contribution to the metric due to the material fields. In view of the Teyssandier coordinate condition, we can write the linear-approximate solution of the higher-derivative equation of gravitational field given by Eq. (2.1) as (Chen *et al.*, 2003)

$$h_{\mu\nu} = h^{(E)}_{\mu\nu} + \psi_{\mu\nu} + \phi\eta_{\mu\nu}, \tag{2.3}$$

where $h^{(E)}_{\mu\nu}$ decribes a massless tensor field, $\psi_{\mu\nu}$ a massive tensor field, and ϕ a massive scalar field. They are respectively the solutions of the following equations

$$\Box h_{\mu\nu}^{(E)} = -2k\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T\right). \tag{2.4}$$

$$\left(\Box + m_1^2\right)\psi_{\mu\nu} = 2k\left(T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}T\right). \tag{2.5}$$

and

$$\left(\Box + m_0^2\right)\phi = -\frac{1}{3}kT, \tag{2.6}$$

where the symbol $\Box$ denotes the d'Alembertion operator with $\Box = \eta^{\alpha\beta}\partial_\alpha\partial_\beta$, $m_0^2 = 1/2(3a + b)$ and $m_1^2 = -b^{-1}$. It is worth mentioning that m_0 and m_1 can be real or imaginary according to the signs of b and $3a + b$. In our next discussion, we will assume that $m_1^2 > 0\,(b < 0)$ and $m_2^2 > 0$ $(3a + b > 0)$, which corresponds to the absence of tachyone (both positive and negative energy) in the dynamical field, in order to assure asymptotic argreement of the theory with Newton's law (Accioly and Azeredo, 2000; Stelle, 1977).

We now proceed to calculate $h_{\mu\nu}^{(E)}$, $\psi_{\mu\nu}$, and ϕ produced by laser pulse. We suppose the higher-power laser pulse propagates along a wave-guide lying in the x axis with a velocity $\upsilon < c$. The essential features of the $\vec{E}$ and $\vec{B}$ fields are then summarized by the expressions (Kapany and Burke, 1972)

$$E_2(\vec{r}, t) = \xi(\vec{r}, t)\ \sin(\omega t - kx), \tag{2.7}$$

$$B_3(\vec{r}, t) = \left(\frac{\upsilon}{c}\right)\frac{\xi(\vec{r}, t)}{c}\ \sin(\omega t - kx), \tag{2.8}$$

$$B_1(\vec{r}, t) = \left[1 - \left(\frac{\upsilon}{c}\right)^2\right]^{1/2}\frac{\xi(\vec{r}, t)}{c}\ \cos(\omega t - kx), \tag{2.9}$$

where $\xi^2(\vec{r}, t)$ denotes the envelope of our pulse . Now, for a "thin" tightly focused pulse of duration T, we may write

$$\xi(\vec{r}, t) = AE_0^2[\theta(\upsilon(t + T) - x) - \theta(\upsilon t - x)]\delta(y)\delta(z), \tag{2.10}$$

where A is the effective cross-sectional area for the laser pulse, E_0 is the pulse amplitude and the step function $\theta(x)$ is defined by $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$.

The energy-momentum tensor for our pulse is

$$T_{\mu\nu} = \frac{1}{2}\varepsilon_0\xi^2(\vec{r}, t)M_{\mu\nu}, \tag{2.11}$$

where

$$M_{\mu\nu} = \begin{pmatrix} 1 & -\frac{\beta}{c} & 0 & 0 \\ -\frac{\beta}{c} & \frac{\beta^2}{c^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\beta^2}{c^2} \end{pmatrix}, \tag{2.12}$$

with $\beta = \upsilon/c$. Substituting $T_{\mu\nu}$ from Eq. (2.11) into Eqs. (2.4), (2.5), and (2.6), and solving the equations, we get

$$h^{(E)}_{\mu\nu} = h^{(E)}_{00} M_{\mu\nu}, \tag{2.13}$$

$$\Psi_{\mu\nu} = \Psi_{00} M_{\mu\nu}, \tag{2.14}$$

and

$$\phi = 0, \tag{2.15}$$

where

$$h^{(E)}_{00} = -\frac{k\rho A}{2\pi} \ln\left(\frac{[x-\upsilon(t+T)] + \{[x-\upsilon(t+T)]^2 + (1-\beta^2)(y^2+z^2)\}^{1/2}}{(x-\upsilon t) + [(x-\upsilon t)^2 + (1-\beta^2)(y^2+z^2)]^{1/2}}\right). \tag{2.16}$$

$$\psi_{00} = \frac{k\rho A}{2\pi} \exp\{-m_1(1-\beta^2)^{-1/2}[(x-\upsilon t)^2 + (1-\beta^2)(y^2+z^2)]^{1/2}\} \times \ln\left(\frac{[x-\upsilon(t+T)] + \{[x-\upsilon(t+T)]^2 + (1-\beta^2)(y^2+z^2)\}^{1/2}}{(x-\upsilon t) + [(x-\upsilon t)^2 + (1-\beta^2)(y^2+z^2)]^{1/2}}\right). \tag{2.17}$$

The radiation energy density ρ appearing in Eqs. (2.16) and (2.17) is defined by

$$\rho = \frac{1}{2}\varepsilon_0 E_0^2. \tag{2.18}$$

Eqs. (2.15) implies that our laser pulse does not produce the massive scalar field ϕ of higher derivative gravity. Inserting Eqs. (2.13), (2.14), and (2.15) into (2.3), we have

$$h_{\mu\nu} = H M_{\mu\nu}, \tag{2.19}$$

where

$$\begin{aligned} H(\vec{r},t) &= h^{(E)}_{00}(\vec{r},t) + \psi_{00}(\vec{r},t). \\ &= -\frac{k\rho A}{2\pi}\{1 - \exp\{-m_1(1-\beta^2)^{-1/2}[(x-\upsilon t)^2 + (1-\beta^2)(y^2+z^2)]^{1/2}\}\} \\ &\times \ln\left(\frac{[x-\upsilon(t+T)] + \{[x-\upsilon(t+T)]^2 + (1-\beta^2)(y^2+z^2)\}^{1/2}}{(x-\upsilon t) + [(x-\upsilon t)^2 + (1-\beta^2)(y^2+z^2)]^{1/2}}\right). \end{aligned} \tag{2.20}$$

Finally, we note that for a short pulse such that

$$\upsilon T/[(x-\upsilon t)^2+(1-\beta^2)(y^2+z^2)]^{1/2}\ll 1, \tag{2.21}$$

Equations (2.16), (2.17), and (2.20) become, respectively

$$h_{00}^{(E)}=-\frac{(4G\rho V/c^2)}{[(x-\upsilon t)^2+(1-\beta^2)(y^2+z^2)]^{1/2}}, \tag{2.22}$$

$$\psi_{00}=-h_{00}^{(E)}\exp\left\{-m_1(1-\beta^2)^{-1/2}[(x-\upsilon t)^2+(1-\beta^2)(y^2+z^2)]^{1/2}\right\}, \tag{2.23}$$

and

$$H=h_{00}^{(E)}\{1-\exp\{-m_1(1-\beta^2)^{-1/2}[(x-\upsilon t)^2+(1-\beta^2)(y^2+z^2)]^{1/2}\}\}, \tag{2.24}$$

where the “volume” of the laser pulse is given by $V = A\upsilon T$

3. RIEMANN CURVATURE OF CURVED SPACETIME

We calculate the Riemann curvature for spacetime to investigate the effect of spacetime curvature on the energy levels of an atom.The curved spacetime caused by the laser pulse is characterized, from Eqs. (2.2) and (2.19), by the line element

$$ds^2=\left(1+\frac{H}{c^2}\right)(cdt)^2-2\beta\frac{H}{c^2}(cdt)dx-\left[(1-\beta^2)\frac{H}{c^2}\right]dx^2$$
$$-dy^2-\left[1-(1-\beta^2)\frac{H}{c^2}\right]dz^2. \tag{3.1}$$

Because of the arbitrariness of the choice of the coordinate system used in the metric theories of gravitation, we can perform a coordinate transformation:

$$\bar{x}=x-\upsilon t,\quad \bar{t}=\frac{1}{1-\beta^2}\left(t-\frac{\beta}{c}x\right),\qquad \bar{y}=y,\ \bar{z}=z. \tag{3.2}$$

The line element expressed in terms of the new coordinates has the diagonal form

$$ds^2=(1-\beta^2)\left[1+(1-\beta^2)\frac{H}{c^2}\right](cd\bar{t})^2-(1-\beta^2)^{-1}(d\bar{x})^2-(d\bar{y})^2$$
$$-\left[1-(1-\beta^2)\frac{H}{c^2}\right](d\bar{z})^2. \tag{3.3}$$

Thus, the nonzero metric components are

$$\bar{g}_{00} = (1-\beta^2)\left[1+(1-\beta^2)\frac{H}{c^2}\right], \quad \bar{g}_{11} = -(1-\beta^2)^{-1},$$
$$\bar{g}_{22} = -1, \quad \bar{g}_{33} = -1+(1-\beta^2)\frac{H}{c^2}, \tag{3.4}$$

where

$$H = h_{00}^{(E)}\{1-\exp\{-m_1(1-\beta^2)^{-1/2}[\bar{x}^2+(1-\beta^2)A_1]^{1/2}\}\} \tag{3.5}$$

with

$$h_{00}^{(E)} = -\frac{(4G\rho V/c^2)}{[\bar{x}^2+(1-\beta^2)A_1]^{1/2}}. \tag{3.6}$$

Here, We have written A_1 for y^2+z^2. Using the metric (3.4) , we find the nonzero components of the Riemann curvature tensor

$$R_{1010} = R_{0101} = -R_{1001} = -R_{0110}$$
$$= \frac{4\bar{x}^2(1-\beta^2)\frac{H}{c^2}+(1-\beta^2)\left[3\bar{x}^2-2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^2-2A_1(1-\beta^2)^4\left(\frac{H}{c^2}\right)^2}{4[\bar{x}^2+(1-\beta^2)A_1]^2\left[1+(1-\beta^2)\frac{H}{c^2}\right]} \tag{3.7}$$

$$R_{3030} = R_{0303} = -R_{3003} = -R_{0330} = -\frac{(1-\beta^2)^4\bar{x}^2\left(\frac{H}{c^2}\right)^2}{4[\bar{x}^2+(1-\beta^2)A_1]^2}, \tag{3.8}$$

$$R_{1313} = R_{3131} = -R_{1331} = -R_{3113}$$
$$= \frac{4\bar{x}^2(1-\beta^2)\frac{H}{c^2}-(1-\beta^2)\left[3\bar{x}^2+2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^2+2A_1(1-\beta^2)^3\left(\frac{H}{c^2}\right)^2}{4[\bar{x}^2+(1-\beta^2)A_1]^2\left[1-(1-\beta^2)\frac{H}{c^2}\right]}. \tag{3.9}$$

The nonzero components of the Ricci tensor are

$$R_{00} = R^1_{010} + R^3_{030}$$
$$= \frac{1}{4[\bar{x}^2+A_1(1-\beta^2)]^2\left[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]}\left\{-4\bar{x}^2(1-\beta^2)^3\frac{H}{c^2}+2(1-\beta^2)^4\right.$$
$$\left.\times\left[\bar{x}^2+A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^2+4\bar{x}^2(1-\beta^2)^5\left(\frac{H}{c^2}\right)^3-2A_1(1-\beta^2)^6\left(\frac{H}{c^2}\right)^3\right\}, \tag{3.10}$$

$$R_{11} = R^0_{101} + R^3_{131}$$
$$= \frac{1}{4[\bar{x}^2 + A_1(1-\beta^2)]^2\left[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]}\left\{-10\bar{x}^2(1-\beta^2)\left(\frac{H}{c^2}\right)^2\right.$$
$$\left. +4A_1(1-\beta^2)^3\left(\frac{H}{c^2}\right)^2 + 6\bar{x}^2(1-\beta^2)^4\left(\frac{H}{c^2}\right)^4 - 4A_1(1-\beta^2)^5\left(\frac{H}{c^2}\right)^4\right\}, \quad (3.11)$$

$$R_{33} = R^0_{303} + R^1_{313}$$
$$= \frac{1}{4[\bar{x}^2 + A_1(1-\beta^2)]^2\left[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]}\left\{-4\bar{x}^2(1-\beta^2)^2\frac{H}{c^2} - 2(1-\beta^2)^3\right.$$
$$\left. \times\left[\bar{x}^2 - A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^2 + 4\bar{x}^2(1-\beta^2)^4\left(\frac{H}{c^2}\right)^3 - 2A_1(1-\beta^2)^5\left(\frac{H}{c^2}\right)^3\right\}. \quad (3.12)$$

The Ricci scalar curvature is given by

$$R = \frac{1}{4[\bar{x}^2 + A_1(1-\beta^2)]^2\left[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]}\left\{22\bar{x}^2(1-\beta^2)^3\left(\frac{H}{c^2}\right)^2\right.$$
$$\left. -8A_1(1-\beta^2)^4\left(\frac{H}{c^2}\right)^2 - 14\bar{x}^2(1-\beta^2)^5\left(\frac{H}{c^2}\right)^4 + 8A_1(1-\beta^2)^6\left(\frac{H}{c^2}\right)^4\right\}. \quad (3.13)$$

4. CURVATURE TENSOR IN FERMI NORMAL COORDINATES

Fermi normal coordinates are appropriate for a problem involving energy levels. In Fermi normal coordinates (Parker, 1980a,b), each spacelike hypersurface of constant x° is generated by the set of spacelike geodesics normal at a point to the timelike geodesics $p(\tau)$ along which the atom is freely falling. The time x° of an event in the hypersurface is the proper time τ along $p(\tau)$ at the point where it intersects the hypersurface. These coordinates are normal along the geodesic $p(\tau)$.

One can now construct a Fermi normal basis in the metric given by Eq. (3.4). The first integrals of motion for a particle moving along the geodesics in the x direction in the metric field are (Ji *et al.*, 1998).

$$c\dot{\bar{t}} = \pm(1-\beta^2)^{1/2}(\bar{g}_{00})^{-1}, \quad \dot{\bar{x}} = \pm(1-\beta^2)^{1/2}[(1-\beta^2)(\bar{g}_{00})^{-1} - 1], \quad (4.1)$$

where the dot above the letter means derivative with respect to proper time τ. Using the above equations, one can verify that a Fermi normal basis for the x geodesic is given by

$$\hat{e}_{\hat{0}} = c\bar{t}\frac{\partial}{c\partial\bar{t}} + \bar{x}\frac{\partial}{\partial\bar{x}}, \quad \hat{e}_{\hat{1}} = \alpha\frac{\partial}{c\partial\bar{t}} + \eta\frac{\partial}{\partial\bar{x}}, \quad \hat{e}_{\hat{2}} = \frac{\partial}{\partial\bar{y}},$$
$$\hat{e}_{\hat{3}} = \left[1 - (1-\beta^2)\frac{H}{c^2}\right]^{-1/2}\frac{\partial}{\partial\bar{z}}, \tag{4.2}$$

where

$$\alpha^2 = -\frac{H/c^2}{\left[1 + (1-\beta^2)\frac{H}{c^2}\right]^2},$$
$$\eta^2 = \frac{1-\beta^2}{1+(1-\beta^2)\frac{H}{c^2}}. \tag{4.3}$$

It is apparent that $(\hat{e}_{\hat{0}})^0 = c\bar{t}$, $(\hat{e}_{\hat{0}})^1 = \bar{x}$, $(\hat{e}_{\hat{1}})^0 = \alpha$, $(\hat{e}_{\hat{1}})^1 = \eta$,

$$(\hat{e}_{\hat{2}})^2 = 1, \quad (\hat{e}_{\hat{3}})^3 = \left[1 - (1-\beta^2)\frac{H}{c^2}\right]^{-1/2}. \tag{4.4}$$

In the Fermi frame, the components of the curvature tensor are given by

$$R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}} = R_{\mu\nu\sigma\rho}(\hat{e}_{\hat{\alpha}})^{\mu}(\hat{e}_{\hat{\beta}})^{\nu}(\hat{e}_{\hat{\gamma}})^{\sigma}(\hat{e}_{\hat{\delta}})^{\rho}, \tag{4.5}$$

where $R_{\mu\nu\sigma\rho}$ are the components of the Riemann tensor in the Riemann normal coordinates. Substituting Eqs. (3.7), (3,8), (3,9), and (4.4) into (5.5) yields the nonzero components of the Riemann tensor in Fermi normal coordinates

$$R_{\hat{1}\hat{0}\hat{1}\hat{0}} = R_{\hat{0}\hat{1}\hat{0}\hat{1}} = -R_{\hat{0}\hat{1}\hat{1}\hat{0}} = -R_{\hat{1}\hat{0}\hat{0}\hat{1}}$$
$$= \frac{4\bar{x}(1-\beta^2)^2\left(\frac{H}{c^2}\right) + (1-\beta^2)^3\left[3\bar{x}^2 - 2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^2 - 2A_1(1-\beta^2)^4\left(\frac{H}{c^2}\right)^2}{4[\bar{x}^2 + A_1(1-\beta^2)]^2\left[1+(1-\beta^2)^2\frac{H}{c^2}\right]^2} \tag{4.6}$$

$$R_{\hat{3}\hat{0}\hat{3}\hat{0}} = R_{\hat{0}\hat{3}\hat{0}\hat{3}} = -R_{\hat{0}\hat{3}\hat{3}\hat{0}} = -R_{\hat{3}\hat{0}\hat{0}\hat{3}}$$
$$= \frac{(2A_1 - 5\bar{x}^2)(1-\beta^2)\left(\frac{H}{c^2}\right)^2 + [3\bar{x}^2 - 2A_1(1-\beta^2)](1-\beta^2)^5\left(\frac{H}{c^2}\right)^4}{4[\bar{x}^2 + A_1(1-\beta^2)]^2\left[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]^2} \tag{4.7}$$

$$R_{\hat{1}\hat{3}\hat{1}\hat{3}} = R_{\hat{3}\hat{1}\hat{3}\hat{1}} = -R_{\hat{1}\hat{3}\hat{3}\hat{1}} = -R_{\hat{3}\hat{1}\hat{1}\hat{3}}$$
$$= \frac{1}{4[\bar{x}^2 + A_1(1-\beta^2)]^2\left[[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]^2}\left\{4\bar{x}^2(1-\beta^2)^2\frac{H}{c^2}\right.$$

$$+(1-\beta^2)^3\left[\overline{x}^2 - 2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^2 - 2\overline{x}^2(1-\beta^2)^4\left(\frac{H}{c^2}\right)^3$$
$$-(1-\beta^2)^5\left[\overline{x}^2 - 2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^4\Bigg\}. \tag{4.8}$$

In Fermi normal coordinates, the components of the Ricci tensor are given by

$$R_{\hat{\alpha}\hat{\beta}} = R_{\mu\nu}(\hat{e}_{\hat{\alpha}})^{\mu}(\hat{e}_{\hat{\beta}})^{\nu}. \tag{4.9}$$

Substituting Eqs. (3.10), (3.11), (3.12), and (4.4) into Eq. (4.9), we obtain the nonzero components of the Ricci tensor

$$R_{\hat{0}\hat{0}} = \frac{1}{4[\overline{x}^2 + A_1(1-\beta^2)]^2\left[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]^2}\Bigg\{-4\overline{x}^2(1-\beta^2)^2\frac{H}{c^2}$$
$$+(1-\beta^2)^3\left[10\overline{x}^2 + 2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^2$$
$$+(1-\beta^2)^4\left[2\overline{x}^2 - 4A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^3$$
$$-(1-\beta^2)^5\left[6\overline{x}^2 + 2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^4 + 4A_1(1-\beta^2)^6\left(\frac{H}{c^2}\right)^5\Bigg\}, \tag{4.10}$$

$$R_{\hat{1}\hat{1}} = \frac{1}{4[\overline{x}^2 + A_1(1-\beta^2)]^2\left[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]^2}\Bigg\{-6\overline{x}^2(1-\beta^2)^3\left(\frac{H}{c^2}\right)^2$$
$$+2A_1(1-\beta^2)^4\left(\frac{H}{c^2}\right)^2 + 4\overline{x}^2(1-\beta^2)^5\left(\frac{H}{c^2}\right)^4 - 2A_1(1-\beta^2)^6\left(\frac{H}{c^2}\right)^4\Bigg\}, \tag{4.11}$$

$$R_{\hat{3}\hat{3}} = \frac{1}{4[\overline{x}^2 + A_1(1-\beta^2)]^2\left[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]^2}\Bigg\{-4\overline{x}^2(1-\beta^2)^2\frac{H}{c^2}$$
$$-(1-\beta^2)^3\left[6\overline{x}^2 - 2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^2$$
$$+(1-\beta^2)^4\left[2\overline{x}^2 + 2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^3$$

$$+(1-\beta^2)^5\left[4\overline{x}^2-2A_1\left(\frac{H}{c^2}\right)^{-1}\right]\left(\frac{H}{c^2}\right)^4-2A_1(1-\beta^2)^6\left(\frac{H}{c^2}\right)^4\Bigg\}. \tag{4.12}$$

Thus, the Ricci scalar curvature is

$$\begin{aligned}R &= R^{\hat{0}}_{\hat{0}}+R^{\hat{1}}_{\hat{1}}+R^{\hat{3}}_{\hat{3}} \\ &= \frac{1}{4[\overline{x}^2+A_1(1-\beta^2)]^2\left[1-(1-\beta^2)^2\left(\frac{H}{c^2}\right)^2\right]^2}\Bigg\{22\overline{x}^2(1-\beta^2)^3\left(\frac{H}{c^2}\right)^2 \\ &\quad -8A_1(1-\beta^2)^4\left(\frac{H}{c^2}\right)^2-14\overline{x}^2(1-\beta^2)^5\left(\frac{H}{c^2}\right)^4+8A_1(1-\beta^2)^6\left(\frac{H}{c^2}\right)^4\Bigg\},\end{aligned} \tag{4.13}$$

which is consistent with that given in Eq. (3.13).

5. ENERGY-LEVEL SHIFTS OF ONE-ELECTRON ATOM

Now we can use the curvature tensors given in the preceding section in Fermi normal coordinates and the expression for gravitational perturbation of the atomic energy-level given by Parker and Pimentel (1982) to calculate explicitly the energy-level shifts of a one-electron atom in the curved spacetime because of the high-power laser pulse. We will investigate the energy-level shifts of an atom being located at $\overline{x}=0$. On account of $|H|/c^2 \ll 1$, we reduce the above curvature and Ricci tensors in Fermi normal coordinates to

$$R_{\hat{0}\hat{1}\hat{0}\hat{1}}=R_{\hat{1}\hat{0}\hat{1}\hat{0}}=-R_{\hat{0}\hat{1}\hat{1}\hat{0}}=-R_{\hat{1}\hat{0}\hat{0}\hat{1}}\approx-\frac{1}{2c^2A_1}(1-\beta^2)h^{(E)}_{00}(1-e^{-m_1\sqrt{A_1}}), \tag{5.1}$$

$$R_{\hat{0}\hat{3}\hat{0}\hat{3}}=R_{\hat{3}\hat{0}\hat{3}\hat{0}}=-R_{\hat{3}\hat{0}\hat{0}\hat{3}}=-R_{\hat{0}\hat{3}\hat{3}\hat{0}}\approx 0, \tag{5.2}$$

$$R_{\hat{1}\hat{3}\hat{1}\hat{3}}=R_{\hat{3}\hat{1}\hat{3}\hat{1}}=-R_{\hat{1}\hat{3}\hat{3}\hat{1}}=-R_{\hat{3}\hat{1}\hat{1}\hat{3}}\approx-\frac{1}{2c^2A_1}(1-\beta^2)h^{(E)}_{00}(1-e^{-m_1\sqrt{A_1}}), \tag{5.3}$$

$$R_{\hat{0}\hat{0}}\approx\frac{1}{2c^2A_1}(1-\beta^2)h^{(E)}_{00}(1-e^{-m_1\sqrt{A_1}}), \tag{5.4}$$

$$R_{\hat{1}\hat{1}}\approx 0, \tag{5.5}$$

$$R_{\hat{3}\hat{3}}\approx\frac{1}{2c^2A_1}(1-\beta^2)h^{(E)}_{00}(1-e^{-m_1\sqrt{A_1}}), \tag{5.6}$$

$$R\approx 0. \tag{5.7}$$

Here

$$h_{00}^{(E)} = -\frac{(4G\rho V/c^2)}{[(1-\beta^2)A_1]^{1/2}}. \tag{5.8}$$

Substituting the above curvature and Ricci tensors into the expression for energy-level shift as follows (Parker and Pimentel, 1982)

$$E^{(1)} = MR_{\hat{0}\hat{0}} + NR + \sum_{i=1}^{3} C^{ii} R_{\hat{o}\hat{i}\hat{o}\hat{i}},$$

we have

$$E^{(1)} = \frac{1}{2c^2 A_1}(1-\beta^2)h_{00}^{(E)}\left(1-e^{-m_1\sqrt{A_1}}\right)(M - C^{11}). \tag{5.9}$$

M, N, and C^{ii} are constants in relationship to the state. For highly excited hydrogen or Rydberg atoms, we have

$$E_n^{(1)} \approx 2.5\frac{\hbar^2}{\alpha^2 m_e}\left[\frac{2G\rho V}{c^4 A_1^{3/2}}(1-\beta^2)^{1/2}\left(1-e^{-m_1\sqrt{A_1}}\right)\right]n^4, \tag{5.10}$$

where α and m_e are respectively the fine structure constant and the electron rest mass. The unperturbed energy of the nth level is

$$E_n^{(0)} = -2\pi\hbar c R_y n^{-2}, \tag{5.11}$$

where R_y is the Rydberg constant. From Eqs. (5.10) and (5.1 l), one finds that for large n

$$D = \frac{E_{n+1}^{(1)} - E_n^{(1)}}{E_{n+1}^{(0)} - E_n^{(0)}} = \tilde{D}\left(1-e^{-m_1\sqrt{A_1}}\right), \tag{5.12}$$

where

$$\tilde{D} = \frac{\tilde{E}_{n+1}^{(1)} - \tilde{E}_n^{(1)}}{E_{n+1}^{(0)} - E_n^{(0)}} \approx 5.26\times 10^{-16}\frac{2G\rho V}{c^4 A_1^{3/2}}(1-\beta^2)^{1/2}n^6 \tag{5.13}$$

is the result in general relativity (Ji *et al*., 1998).

In view of the fact that the output energy of high-intensity laser pulse can come up to 10^6 J, we let $\rho V = 2.5\times 10^6$ J. Putting this into Eq. (5.13), and taking $\upsilon = 0.9$ c, $A_1 = 10^{-12}$ m^2, and $n = 10^2$, we have

$$\tilde{D} = \frac{\Delta\tilde{E}_n^{(1)}}{\Delta E_n^{(0)}} \approx 10^{-24}. \tag{5.14}$$

This precision is attainable in experimental observation at present (Scully, 1979; Ji *et al.*, 1998).

We now proceed to make a numerical comparison between the results predicted by higher-derivative theory of gravitation and prediction of genera1 relativity. Consider the case of the force ranges of the Yukawa-type corrections to the Newtonian component $m_1^{-1} = \beta^{-1} < 1$ cm (Fujii, 1971; Mikkelson and Newman, 1977). For the purpose of clearness, we find some typical values, such as $D \sim \tilde{D}$ for $m_1^{-1} \leq 10^{-7}$ m, $D \sim 0.631\tilde{D}$ for $m_1^{-1} = 10^{-6}$ m, $D \sim 0.100\tilde{D}$ for $m_1^{-1} = 10^{-5}$ m, $D \sim 0.010\tilde{D}$ for $m_1^{-1} = 10^{-4}$ m, $D \sim 0.002\tilde{D}$ for $m_1^{-1} = 10^{-3}$ m. The confrontation of these results with the probable measurement values should test higher-derivative theory of gravitation and estimate the range of additional force m_1^{-1}.

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